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Dynamical generation of three flavor mixing

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Abstract. In this paper we study patterns of dynamical symmetry breaking for a class of models characterized by $SU(3)_V \times SU(3)_A \times U(1)_V$ chiral-flavor symmetry. In particular, we show how dynamical mixing generation implies the existence of exotic condensates on vacuum, mixing fermion and antifermions with different flavors. By means of Ward–Takahashi identities we also derive Haag expansions for Nambu–Goldstone fields.

1. Introduction

Dynamical generation of masses and mixing represents an important subject in quantum chromodynamics [1, 2] and in the physics beyond the Standard model [2, 3].

The study of mixing transformation in the quantum field theoretic (QFT) framework [4, 5] reveals new features in comparison with the standard quantum mechanical treatment [6, 7, 8]. This allows to derive a neutrino oscillation formula which is valid at all energy scales [9]. The difference between the latter *exact* oscillation formula and the standard (Pontecorvo) one can be retraced to a complicated condensate structure of the vacuum for flavor fields — the so-called *flavor vacuum*.

It was noticed [10] that a similar vacuum structure is dynamically generated by a four-fermion effective interaction. This is a consequence of the scattering of strings with the brane defects of spacetime, in a string-theory inspired framework. Analogous attempts were also carried out in Ref. [11].

In Ref. [12], it was shown that exotic vacuum condensates provide a necessary condition for a dynamically generated mixing in models with $SU(2)_V \times SU(2)_A \times U(1)_V$ chiral-flavor symmetry. This result was obtained through algebraic reasonings and so it has a manifestly non-perturbative character. Moreover, in the mean-field approximation, one can prove that the ground state for the dynamically-generated mixing structurally resembles the flavor vacuum, in agreement with Ref. [10]. In addition, by employing the Ward–Takahashi (WT) identities and Umezawa's ε -term prescription [13, 14], one can also address the issue of Nambu–Goldstone (NG) modes and their proper counting.



In this paper we extend some of the aforementioned topics to the case of three flavors. In doing so, we study a class of models characterized by $SU(3)_V \times SU(3)_A \times U(1)_V$ chiral-flavor symmetry. We review the explicit symmetry breaking schemes appearing due to the addition of a mass term to the Lagrangian. The interpretation of these results in terms of flavor charges is proposed, in analogy with Ref. [15]. Once more, by studying analogous patterns of spontaneous symmetry breaking (SSB) we will see that dynamically generated mixing requires the presence of exotic condensates on vacuum and we show how NG modes are related with fermion-antifermion bound states through dynamical maps [14].

The paper is organized as it follows: in Section 2 we review different patterns of explicit symmetry breaking and the definition of flavor charges. In Section 3 we show the corresponding patterns of dynamical symmetry breaking and we show that mixed condensates furnish a necessary condition for dynamical mixing generation. Finally in Section 4 we present conclusions and perspectives.

2. Three flavor charges and currents

Let us consider the Lagrangian density \mathcal{L} which is invariant under the global *chiral-flavor* group $G = SU(3)_A \times SU(3)_V \times U(1)_V$. Let the fermion field be a flavor triplet

$$\psi = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix}. \quad (1)$$

Under a chiral-group transformation \mathbf{g} , we get

$$\psi \rightarrow \psi' = \mathbf{g}\psi = \exp \left[i \left(\phi + \omega \cdot \frac{\lambda}{2} + \omega_5 \cdot \frac{\lambda}{2} \gamma_5 \right) \right] \psi, \quad (2)$$

where $\lambda_j, j = 1, \dots, 8$ are the Gell-Mann matrices and ϕ, ω, ω_5 are real-valued transformation parameters of G . Noether's theorem implies the conserved currents

$$J^\mu = \bar{\psi} \gamma^\mu \psi, \quad \mathbf{J}^\mu = \bar{\psi} \gamma^\mu \frac{\lambda}{2} \psi, \quad \mathbf{J}_5^\mu = \bar{\psi} \gamma^\mu \gamma_5 \frac{\lambda}{2} \psi, \quad (3)$$

and the corresponding conserved charges

$$Q = \int d^3x \psi^\dagger \psi, \quad \mathbf{Q} = \int d^3x \psi^\dagger \frac{\lambda}{2} \psi, \quad \mathbf{Q}_5 = \int d^3x \psi^\dagger \frac{\lambda}{2} \gamma_5 \psi. \quad (4)$$

From these we recover the Lie algebra of the chiral-flavor group G , i.e.

$$\begin{aligned} [Q_i, Q_j] &= i f_{ijk} Q_k, \quad [Q_i, Q_{5,j}] = i f_{ijk} Q_{5,k}, \\ [Q_{5,i}, Q_{5,j}] &= i f_{ijk} Q_k, \quad [Q, Q_{5,j}] = [Q, Q_j] = 0. \end{aligned} \quad (5)$$

Here $i, j, k = 1, \dots, 8$ and f_{ijk} are the structure constants of the $su(3)$ algebra.

For massless fermions the Lagrangian is invariant under both the flavor and axial flavor transformations. The chiral symmetry is explicitly broken when a mass term $\mathcal{L}_M = -\bar{\psi} M \psi$ is added to \mathcal{L} . In fact, one can easily verify, that [2]

$$\partial_\mu J^\mu = 0, \quad \partial_\mu \mathbf{J}^\mu = \frac{i}{2} \bar{\psi} [M, \lambda] \psi, \quad \partial_\mu \mathbf{J}_5^\mu = \frac{i}{2} \bar{\psi} \gamma_5 \{M, \lambda\} \psi. \quad (6)$$

Note, that these relations do not depend on the specific form of the original action. We now prove how different choices of M can affect the structure of the residual (unbroken) subgroup H .

For future convenience it useful to introduce the following notation:

$$\Phi_k = \bar{\psi} \lambda_k \psi, \quad \Phi_k^5 = \bar{\psi} \lambda_k \gamma_5 \psi, \quad \lambda_0 \equiv \mathbb{I}, \quad k = 0, \dots, 8. \quad (7)$$

i) Let $M = m_0 \lambda_0$, then (6) reduces to

$$\partial_\mu J^\mu = \partial_\mu \mathbf{J}^\mu = 0, \quad \partial_\mu \mathbf{J}_5^\mu = i m_0 \Phi^5, \quad (8)$$

i.e., the scalar and vector currents remain conserved and the broken-phase symmetry is $H = U(3)_V$ [2].

ii) If $M = m_0 \lambda_0 + m_3 \lambda_3 + m_8 \lambda_8$, then the mass matrix assumes the form

$$M = \begin{bmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{bmatrix}, \quad (9)$$

where

$$m_e = m_0 + m_3 + \frac{1}{\sqrt{3}} m_8, \quad m_\mu = m_0 - m_3 + \frac{1}{\sqrt{3}} m_8, \quad m_\tau = m_0 - \frac{2}{\sqrt{3}} m_8. \quad (10)$$

The current divergences read

$$\partial_\mu J^\mu = \partial_\mu J_3^\mu = \partial_\mu J_8^\mu = 0, \quad \partial_\mu J_1^\mu = -m_3 \Phi_2, \quad \partial_\mu J_2^\mu = m_3 \Phi_1, \quad (11)$$

$$\partial_\mu J_4^\mu = -\frac{m_3 + \sqrt{3} m_8}{2} \Phi_5, \quad \partial_\mu J_5^\mu = \frac{m_3 + \sqrt{3} m_8}{2} \Phi_4, \quad (12)$$

$$\partial_\mu J_6^\mu = \frac{m_3 - \sqrt{3} m_8}{2} \Phi_7, \quad \partial_\mu J_7^\mu = -\frac{m_3 - \sqrt{3} m_8}{2} \Phi_6, \quad (13)$$

$$\partial_\mu J_{5,k}^\mu = i \left(m_0 + \frac{m_8}{\sqrt{3}} \right) \Phi_k^5, \quad k = 1, 2, \quad (14)$$

$$\partial_\mu J_{5,3}^\mu = i \left[\left(m_0 + \frac{m_8}{\sqrt{3}} \right) \Phi_3^5 + \frac{m_3}{\sqrt{3}} \left(\Phi_8^5 + \frac{2}{\sqrt{3}} \Phi_0^5 \right) \right], \quad (15)$$

$$\partial_\mu J_{5,k}^\mu = i \left[m_0 + \frac{1}{2} \left(m_3 - \frac{m_8}{\sqrt{3}} \right) \right] \Phi_k^5, \quad k = 4, 5, \quad (16)$$

$$\partial_\mu J_{5,k}^\mu = i \left[m_0 - \frac{1}{2} \left(m_3 + \frac{m_8}{\sqrt{3}} \right) \right] \Phi_k^5, \quad k = 6, 7, \quad (17)$$

$$\partial_\mu J_{5,8}^\mu = i \left[\left(m_0 - \frac{m_8}{\sqrt{3}} \right) \Phi_8^5 + \frac{m_3}{\sqrt{3}} \Phi_3^5 + \frac{2m_8}{3} \Phi_0^5 \right]. \quad (18)$$

The residual symmetry is thus $U(1)_V \times U(1)_V^3 \times U(1)_V^8$.

iii) If $M = \sum_{k=0}^8 \lambda_k m_k$, we can write

$$M = \begin{bmatrix} m_e & m_{e\mu} e^{i\delta_{e\mu}} & m_{e\tau} e^{i\delta_{e\tau}} \\ m_{e\mu} e^{-i\delta_{e\mu}} & m_\mu & m_{\mu\tau} e^{i\delta_{\mu\tau}} \\ m_{e\tau} e^{-i\delta_{e\tau}} & m_{\mu\tau} e^{-i\delta_{\mu\tau}} & m_\tau \end{bmatrix}, \quad (19)$$

where

$$m_{e\mu} e^{i\delta_{e\mu}} = m_1 - i m_2, \quad m_{e\tau} e^{i\delta_{e\tau}} = m_4 - i m_5, \quad m_{\mu\tau} e^{i\delta_{\mu\tau}} = m_6 - i m_7. \quad (20)$$

As known, two phases can be reabsorbed with an opportune redefinition of fields [2, 17]. One would be thus tempted to put two of the three phases equal to zero (which is common in literature). Only one CP -violating phase would thus remain [2, 16, 17]. However, such “unphysical” phases were precious in the two-flavor case [12] in order to get the correct counting of NG modes.

The current divergences now read:

$$\partial_\mu J_1^\mu = g_{23} + \frac{1}{2}(g_{47} + g_{65}) , \quad (21)$$

$$\partial_\mu J_2^\mu = g_{31} + \frac{1}{2}(g_{46} + g_{57}) , \quad (22)$$

$$\partial_\mu J_3^\mu = g_{12} + \frac{1}{2}(g_{45} + g_{76}) , \quad (23)$$

$$\partial_\mu J_4^\mu = \frac{1}{2}(g_{71} + g_{62} + g_{53} + \sqrt{3}g_{58}) , \quad (24)$$

$$\partial_\mu J_5^\mu = \frac{1}{2}(g_{16} + g_{72} + g_{34} + \sqrt{3}g_{84}) , \quad (25)$$

$$\partial_\mu J_6^\mu = \frac{1}{2}(g_{51} + g_{24} + \sqrt{3}g_{78} + g_{37}) , \quad (26)$$

$$\partial_\mu J_7^\mu = \frac{1}{2}(g_{14} + g_{25} + \sqrt{3}g_{86} + g_{63}) , \quad (27)$$

$$\partial_\mu J_8^\mu = \frac{\sqrt{3}}{2}(g_{45} + g_{67}) , \quad \partial_\mu J^\mu = 0 , \quad (28)$$

$$\partial_\mu J_{5,1}^\mu = i m_0 \Phi_1^5 + \frac{2i}{3} m_1 \Phi_0^5 + \frac{i}{\sqrt{3}} h_{18} + \frac{i}{2}(h_{46} + h_{57}) , \quad (29)$$

$$\partial_\mu J_{5,2}^\mu = i m_0 \Phi_2^5 + \frac{2i}{3} m_2 \Phi_0^5 + \frac{i}{\sqrt{3}} h_{28} + \frac{i}{2}(h_{56} - h_{47}) , \quad (30)$$

$$\partial_\mu J_{5,3}^\mu = i m_0 \Phi_3^5 + \frac{2i}{3} m_3 \Phi_0^5 + \frac{i}{\sqrt{3}} h_{38} + \frac{i}{2}(h_{45} - h_{67}) , \quad (31)$$

$$\partial_\mu J_{5,4}^\mu = i m_0 \Phi_4^5 + \frac{2i}{3} m_4 \Phi_0^5 - \frac{i}{2\sqrt{3}} h_{48} + \frac{i}{2}(h_{16} - h_{27} + h_{34}) , \quad (32)$$

$$\partial_\mu J_{5,5}^\mu = i m_0 \Phi_5^5 + \frac{2i}{3} m_5 \Phi_0^5 - \frac{i}{2\sqrt{3}} h_{58} + \frac{i}{2}(h_{17} + h_{26} + h_{35}) , \quad (33)$$

$$\partial_\mu J_{5,6}^\mu = i m_0 \Phi_6^5 + \frac{2i}{3} m_6 \Phi_0^5 - \frac{i}{2\sqrt{3}} h_{68} + \frac{i}{2}(h_{14} + h_{25} - h_{36}) , \quad (34)$$

$$\partial_\mu J_{5,7}^\mu = i m_0 \Phi_7^5 + \frac{2i}{3} m_7 \Phi_0^5 - \frac{i}{2\sqrt{3}} h_{78} + \frac{i}{2}(h_{15} + h_{24} - h_{37}) , \quad (35)$$

$$\partial_\mu J_{5,8}^\mu = i m_0 \Phi_8^5 + \frac{2i}{3} m_8 \Phi_0^5 + \frac{i}{\sqrt{3}} \left[\Phi_1^5 + \Phi_2^5 + \Phi_3^5 + \frac{1}{2}(\Phi_4^5 + \Phi_5^5 - \Phi_6^5 - \Phi_7^5) - 2\Phi_8^5 \right] . \quad (36)$$

where

$$g_{ij} \equiv m_i \Phi_j - m_j \Phi_i , \quad h_{ij} \equiv m_i \Phi_j^5 + m_j \Phi_i^5 . \quad (37)$$

The residual symmetry is then $H = U(1)_V$. The rôle of this symmetry can be understood as it follows: We introduce the *flavor-charges* [15]

$$Q_e \equiv \frac{1}{3}Q + Q_3 + \frac{1}{\sqrt{3}}Q_8, \quad Q_\mu \equiv \frac{1}{3}Q - Q_3 + \frac{1}{\sqrt{3}}Q_8, \quad Q_\tau \equiv \frac{1}{3}Q - \frac{2}{\sqrt{3}}Q_8, \quad (38)$$

where the total flavor-charge $Q = Q_e + Q_\mu + Q_\tau$. In absence of mixing (i.e., in the cases i) and ii)), Q_e , Q_μ and Q_τ are separately conserved. However, when mixing among the three generations is present, the residual symmetry is the phase symmetry connected with the conservation of the total flavor charge Q .

3. Dynamical generation of masses and mixing

From the considerations of the previous section, we figure out that field mixing is dynamically generated via the SSB scheme

$$SU(3)_A \times SU(3)_V \times U(1)_V \longrightarrow U(1)_V. \quad (39)$$

Let us recall [2, 13, 17] that SSB is characterized by the existence of some local operator(s) ϕ so that on the vacuum $|\Omega\rangle$.

$$\langle [N_i, \phi(0)] \rangle = \langle \varphi_i(0) \rangle \equiv v_i \neq 0, \quad (40)$$

where $\langle \dots \rangle \equiv \langle \Omega | \dots | \Omega \rangle$. v_i are named *order parameters* and N_i represent group generators from the quotient space G/H , where H is the *vacuum stability subgroup*. In our case N_i will be given by \mathbf{Q} and \mathbf{Q}_5 according to the SSB scheme considered and H will coincide with the residual subgroup of the previous section. Moreover, in order to discuss the NG modes it is convenient to introduce in the Lagrangian density a symmetry-breaking term via the so-called ε -term prescription [13, 14] as

$$\mathcal{L}_\varepsilon = \varepsilon \Phi. \quad (41)$$

At the end of calculations the limit $\varepsilon \rightarrow 0$ has to be taken. We thus employ the WT identity in the form [13, 14]

$$i \langle \delta_{(5),k} \psi(0) \rangle = \int d^4y \langle T [\delta_{(5),k} \mathcal{L}(y) \psi(0)] \rangle, \quad (42)$$

which is valid for any local operator $\psi(x)$. In particular we will choose $\psi = \delta_{(5),k} \Phi$, so that:

$$i \langle \delta_{(5),k}^2 \Phi(0) \rangle = \lim_{\varepsilon \rightarrow 0} \varepsilon \int d^4y \langle T [\delta_{(5),k} \Phi(y) \delta_{(5),k} \Phi(0)] \rangle. \quad (43)$$

If the LHS differs from zero, the dynamical map (or Haag expansion) [13, 14, 18] of $\delta_{(5),k} \Phi$ has to contain massless fields as linear terms, i.e the NG fields. This is the statement of the Goldstone theorem [2, 13, 14, 17].

Let us now analyze the three SSB schemes $G \rightarrow H$, in analogy with the previous section.

i) SSB sequence corresponding to a single mass generation is

$$SU(3)_A \times SU(3)_V \times U(1)_V \longrightarrow SU(3)_V \times U(1)_V \sim U(3)_V. \quad (44)$$

The order parameters are

$$\langle \Phi_0 \rangle = v_0 \neq 0, \quad \langle \Phi_k \rangle = 0, \quad k = 1, \dots, 8. \quad (45)$$

We take the ε -term in the form:

$$\mathcal{L}_\varepsilon = \varepsilon \Phi_0, \quad (46)$$

so that Eq.(43) reads

$$i\frac{2}{3}v_0 = \lim_{\varepsilon \rightarrow 0} \varepsilon \int d^4y \langle T [\Phi_k^5(y) \Phi_k^5(0)] \rangle, \quad k = 1, \dots, 8. \quad (47)$$

Note that we have eight NG fields which equals to $\dim(G/H)$. Explicitly, we can write the dynamical maps:

$$\Phi_k^5(x) = \sqrt{Z_{\varphi_k^5}} \varphi_k^5(x) + \dots, \quad k = 1, \dots, 8. \quad (48)$$

where φ_k^5 are the NG fields, $Z_{\varphi_k^5}$ are the field renormalization constants and the dots denote higher order terms in the Haag expansions.

ii) SSB sequence corresponding to dynamical generation of different masses is

$$SU(3)_A \times SU(3)_V \times U(1)_V \longrightarrow U(1)_V \times U(1)_V^3 \times U(1)_V^8. \quad (49)$$

The order parameters are

$$\langle \Phi_0 \rangle = v_0 \neq 0, \quad \langle \Phi_3 \rangle = v_3 \neq 0, \quad \langle \Phi_8 \rangle = v_8 \neq 0. \quad (50)$$

However, we can also restrict to the case $v_8 = 0$, as it clear from Eqs.(11)-(13). This assumption simplifies the formulas without affecting the main reasoning. The ε -term is now

$$\mathcal{L}_\varepsilon = \varepsilon (\Phi_0 + \Phi_3). \quad (51)$$

Then one can derive the following WT identities:

$$iv_3 = - \lim_{\varepsilon \rightarrow 0} \varepsilon \int d^4y \langle T [\Phi_k(y) \Phi_k(0)] \rangle, \quad k = 1, 2, 4, 5, 6, 7, \quad (52)$$

$$iv_0 = \lim_{\varepsilon \rightarrow 0} \varepsilon \int d^4y \langle T [\Phi_k^5(y) \Phi_k^5(0)] \rangle, \quad k = 1, 2, \quad (53)$$

$$i \left(\frac{2}{3}v_0 + v_3 \right) = \lim_{\varepsilon \rightarrow 0} \varepsilon \int d^4y \langle T \left[\left(\Phi_3^5(y) + \Phi_0^{5,12}(y) \right) \left(\Phi_3^5(0) + \Phi_0^{5,12}(0) \right) \right] \rangle, \quad (54)$$

$$i \left(v_0 + \frac{3}{4}v_3 \right) = \lim_{\varepsilon \rightarrow 0} \varepsilon \int d^4y \langle T [\Phi_k^5(y) \Phi_k^5(0)] \rangle, \quad k = 4, 5, \quad (55)$$

$$i \left(\frac{4}{3}v_0 - v_3 \right) = \lim_{\varepsilon \rightarrow 0} \varepsilon \int d^4y \langle T [\Phi_k^5(y) \Phi_k^5(0)] \rangle, \quad k = 6, 7, \quad (56)$$

$$i \frac{1}{3} (2v_0 + v_3) = \lim_{\varepsilon \rightarrow 0} \varepsilon \int d^4y \langle T \left[\left(\Phi_8^5(y) + \frac{1}{\sqrt{3}} \Phi_3^5(y) \right) \left(\Phi_8^5(0) + \frac{1}{\sqrt{3}} \Phi_3^5(0) \right) \right] \rangle, \quad (57)$$

where

$$\Phi_0^{5,12} \equiv \sum_{k=1}^2 \bar{\psi}_k \gamma_5 \psi_k = \frac{2}{3} \Phi_0^5 + \frac{1}{\sqrt{3}} \Phi_8^5. \quad (58)$$

These relations teaches us that we have 14 NG modes, which equals $\dim(G/H)$. We can thus write down the NG dynamical maps

$$\Phi_k(x) = \sqrt{Z_{\varphi_k}} \varphi_k(x) + \dots, \quad (59)$$

$$\Phi_k^5(x) = \sqrt{Z_{\varphi_k^5}} \varphi_k^5(x) + \dots, \quad k = 1, 2, 4, 5, 6, 7, \quad (60)$$

$$\Phi_8^5(x) + \frac{1}{\sqrt{3}} \Phi_3^5(x) = \sqrt{Z_{\varphi_{38}^5}} \varphi_{38}^5(x) + \dots, \quad (61)$$

$$\Phi_0^{5,12}(x) + \Phi_3^5(x) = \sqrt{Z_{\varphi_{03}^5}} \varphi_{03}^5(x) + \dots. \quad (62)$$

iii) SSB sequence corresponding to dynamical mixing generation is

$$SU(3)_A \times SU(3)_V \times U(1)_V \longrightarrow U(1)_V. \quad (63)$$

The order parameters are

$$\langle \Phi_k \rangle = v_k \neq 0, \quad k = 0, \dots, 8. \quad (64)$$

Note that, in order to obtain the correct pattern of symmetry breaking, we have

$$\langle \bar{\psi}_i \psi_j \rangle \neq \langle \bar{\psi}_j \psi_i \rangle \neq 0, \quad i \neq j. \quad (65)$$

Therefore, as it was proved in the two flavor case, in Ref. [12], we find that the presence of mixed condensates on vacuum represents a *necessary* condition for dynamical mixing generation. This feature resembles the analogue result obtained within free theory of neutrino mixing in QFT [15]. Moreover, it confirms the specific achievements of Ref. [10] about four-fermion interaction in a string-inspired framework. We remark that our result is non-perturbative because it is only based on symmetry and algebraic considerations.

To discuss NG modes, we would take a ε -term of the form:

$$\mathcal{L}_\varepsilon = \sum_{k=0}^8 \varepsilon \Phi_k. \quad (66)$$

The corresponding WT identities (43) now assume a very long and not illuminating form. We thus report the dynamical maps which contain NG fields:

$$\Phi_2 - \Phi_3 + \frac{1}{2} (\Phi_4 - \Phi_7 + \Phi_6 - \Phi_5) = V_1 + \sqrt{Z_1} \varphi_1 + \dots, \quad (67)$$

$$\Phi_3 - \Phi_1 + \frac{1}{2} (\Phi_4 - \Phi_6 + \Phi_5 - \Phi_7) = V_2 + \sqrt{Z_2} \varphi_2 + \dots, \quad (68)$$

$$\Phi_1 - \Phi_2 + \frac{1}{2} (\Phi_4 - \Phi_5 + \Phi_7 - \Phi_6) = V_3 + \sqrt{Z_3} \varphi_3 + \dots, \quad (69)$$

$$\Phi_7 - \Phi_1 + \Phi_6 - \Phi_2 + \Phi_5 - \Phi_3 + \sqrt{3}(\Phi_5 - \Phi_8) = V_4 + \sqrt{Z_4} \varphi_4 + \dots, \quad (70)$$

$$\Phi_1 - \Phi_6 + \Phi_7 - \Phi_2 + \Phi_3 - \Phi_4 + \sqrt{3}(\Phi_8 - \Phi_4) = V_5 + \sqrt{Z_5} \varphi_5 + \dots, \quad (71)$$

$$\Phi_5 - \Phi_1 + \Phi_2 - \Phi_4 + \sqrt{3}(\Phi_7 - \Phi_8) + \Phi_3 - \Phi_7 = V_6 + \sqrt{Z_6} \varphi_6 + \dots, \quad (72)$$

$$\Phi_1 - \Phi_4 + \Phi_2 - \Phi_5 + \sqrt{3}(\Phi_8 - \Phi_6) + \Phi_6 - \Phi_3 = V_7 + \sqrt{Z_7}\varphi_7 + \dots, \quad (73)$$

$$\Phi_4 - \Phi_5 + \Phi_6 - \Phi_7 = V_8 + \sqrt{Z_8}\varphi_8 + \dots, \quad (74)$$

$$\Phi_1^5 + \frac{2}{3}\Phi_0^5 + \frac{1}{\sqrt{3}}(\Phi_1^5 + \Phi_8^5) + \frac{1}{2}\sum_{k=4}^7\Phi_k^5 = \sqrt{Z_1^5}\varphi_1^5 + \dots, \quad (75)$$

$$\Phi_2^5 + \frac{2}{3}\Phi_0^5 + \frac{1}{\sqrt{3}}(\Phi_2^5 + \Phi_8^5) + \frac{1}{2}(\Phi_5^5 + \Phi_6^5 - \Phi_4^5 - \Phi_7^5) = \sqrt{Z_2^5}\varphi_2^5 + \dots, \quad (76)$$

$$\Phi_3^5 + \frac{2}{3}\Phi_0^5 + \frac{1}{\sqrt{3}}(\Phi_3^5 + \Phi_8^5) + \frac{1}{2}(\Phi_4^5 + \Phi_5^5 - \Phi_6^5 - \Phi_7^5) = \sqrt{Z_3^5}\varphi_3^5 + \dots, \quad (77)$$

$$\Phi_4^5 + \frac{2}{3}\Phi_0^5 - \frac{1}{2\sqrt{3}}(\Phi_4^5 + \Phi_8^5) + \frac{1}{2}(\Phi_1^5 - \Phi_2^5 + \Phi_3^5 + \Phi_4^5 + \Phi_6^5 - \Phi_7^5) = \sqrt{Z_4^5}\varphi_4^5 + \dots \quad (78)$$

$$\Phi_5^5 + \frac{2}{3}\Phi_0^5 - \frac{1}{2\sqrt{3}}(\Phi_5^5 + \Phi_8^5) + \frac{1}{2}(\Phi_1^5 + \Phi_2^5 + \Phi_3^5 + \Phi_5^5 + \Phi_6^5 + \Phi_7^5) = \sqrt{Z_5^5}\varphi_5^5 + \dots \quad (79)$$

$$\Phi_6^5 + \frac{2}{3}\Phi_0^5 - \frac{1}{2\sqrt{3}}(\Phi_6^5 + \Phi_8^5) + \frac{1}{2}(\Phi_1^5 + \Phi_2^5 - \Phi_3^5 + \Phi_4^5 + \Phi_5^5 - \Phi_6^5) = \sqrt{Z_6^5}\varphi_6^5 + \dots \quad (80)$$

$$\Phi_7^5 + \frac{2}{3}\Phi_0^5 - \frac{1}{2\sqrt{3}}(\Phi_7^5 + \Phi_8^5) + \frac{1}{2}(\Phi_1^5 + \Phi_2^5 - \Phi_3^5 + \Phi_4^5 + \Phi_5^5 - \Phi_7^5) = \sqrt{Z_7^5}\varphi_7^5 + \dots \quad (81)$$

$$\Phi_8^5 + \frac{2}{3}\Phi_0^5 + \frac{1}{\sqrt{3}}\left[\Phi_1^5 + \Phi_2^5 + \Phi_3^5 + \frac{1}{2}(\Phi_4^5 + \Phi_5^5 - \Phi_6^5 - \Phi_7^5) - 2\Phi_8^5\right] = \sqrt{Z_8^5}\varphi_8^5 + \dots, \quad (82)$$

where

$$V_1 = v_2 - v_3 + \frac{1}{2}(v_4 - v_7 + v_6 - v_5), \quad (83)$$

$$V_2 = v_3 - v_1 + \frac{1}{2}(v_4 - v_6 + v_5 - v_7), \quad (84)$$

$$V_3 = v_1 - v_2 + \frac{1}{2}(v_4 - v_5 + v_7 - v_6), \quad (85)$$

$$V_4 = v_7 - v_1 + v_6 - v_2 - v_3 - \sqrt{3}v_8 + v_5(1 + \sqrt{3}), \quad (86)$$

$$V_5 = v_1 - v_6 + v_7 - v_2 + v_3 + \sqrt{3}v_8 - v_4(1 + \sqrt{3}), \quad (87)$$

$$V_6 = v_5 - v_1 + v_2 - v_4 + v_3 - (1 - \sqrt{3})v_7 - \sqrt{3}v_8, \quad (88)$$

$$V_7 = v_1 - v_4 + v_2 - v_5 - v_3 + (1 - \sqrt{3})v_6 + \sqrt{3}v_8, \quad (89)$$

$$V_8 = v_4 - v_5 + v_6 - v_7. \quad (90)$$

We thus have 16 independent NG fields, whose number coincides with $\dim(G/H)$.

4. Conclusions

In this paper we have extended some of the results of Ref. [12], to the case of three flavors. We studied three chiral symmetry breaking schemes responsible for: i) dynamical mass generation ii) dynamical generation of different masses iii) dynamical mixing generation. In the last case we recognized that condensates which mix fermion and antifermions of different flavors necessary appear in the vacuum. This ground state structure formally reminds the one encountered in Ref. [15] for the flavor vacuum, in the context of the free theory of flavor mixing. Moreover, this is in agreement with the results of Refs. [10, 11] on dynamical mixing generation induced by an effective four-fermion interaction. The counting of NG field was also carried out, by showing that, at each step, their number coincide with the dimension of the quotient space G/H .

In Ref. [12] the formal structure of flavor vacuum was clearly recognized, when mixing is dynamically generated, with the use of mean field approximation. In this approximation, in fact, one can explicitly write the vacuum in terms of the generators of *Bogoliubov transformations* [2, 19, 20]. The result for dynamical mixing generation was derived by studying vacuum manifolds. A similar analysis should be thus performed in the present case, in order to show the complete analogy with Ref. [15].

It is known since a long time [14, 20, 21], that SSB is intimately related to the dynamical rearrangement of symmetry: the original symmetry group of the Lagrangian is rearranged, at level of the physical fields representation, in a new symmetry group which turns out to be the Inönü–Wigner contraction [22] of the original one. In the present case, the group contraction in the various steps of chiral symmetry breaking should be studied, in order to obtain further informations on physical fields representation.

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